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FRACTIONAL INTEGRAL ASSOCIATED WITH SCHRÖDINGER OPERATOR ON VANISHING GENERALIZED MORREY SPACES

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Abstract. Let $L=-\triangle+V$ be a Schrödinger operator, where the non-negative potential V belongs to the reverse Hölder class $RH_{n/2}$, let b belong to a new $BMO_\theta(\rho)$ space, and let $\mathscr{I}_\beta^L$ be the fractional integral operator associated with L. In this paper, we study the boundedness of the operator $\mathscr{I}_\beta^L$ and its commutators $[b,\mathscr{I}_\beta^L]$ with $b\in BMO_\theta(\rho)$ on generalized Morrey spaces associated with Schrödinger operator $M_{p,\varphi}^{\alpha,V}$ and vanishing generalized Morrey spaces associated with Schrödinger operator $VM_{p,\varphi}^{\alpha,V}$. We find the sufficient conditions on the pair (φ_1,φ_2) which ensures the boundedness of the operator $\mathscr{I}_\beta^L$ from $M_{p,\varphi_1}^{\alpha,V}$ to $M_{q,\varphi_2}^{\alpha,V}$ and from $VM_{p,\varphi_1}^{\alpha,V}$ to $VM_{q,\varphi_2}^{\alpha,V}$, $1/p-1/q=\beta/n$. When b belongs to $BMO_\theta(\rho)$ and (φ_1,φ_2) satisfies some conditions, we also show that the commutator operator $[b,\mathscr{I}_\beta^L]$ is bounded from $M_{p,\varphi_1}^{\alpha,V}$ to $M_{q,\varphi_2}^{\alpha,V}$ and from $VM_{p,\varphi_1}^{\alpha,V}$ to $VM_{q,\varphi_2}^{\alpha,V}$, $1/p-1/q=\beta/n$.

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