Journal of Mathematical Inequalities
Volume 12, Number 3 (2018), 861–872
doi:10.7153/jmi-2018-12-64

BERNSTEIN–TYPE OPERATORS THAT REPRODUCE EXPONENTIAL FUNCTIONS

ALI ARAL, DANIEL CÁRDENAS-MORALES AND PEDRO GARRANCHO

Abstract. In this paper we recover a generalization of the classical Bernstein operators introduced by Morigi and Neamtu in 2000. Specifically, we focus on a sequence of operators that reproduce the exponential functions $\exp(\mu t)$ and $\exp(2\mu t)$, $\mu > 0$. We study its convergence, this including qualitative and quantitative theorems, an asymptotic formula and saturation results. We also show their shape preserving properties by considering generalized convexity. Finally, a comparison is stated, that shows that in a certain sense and for certain family of illustrative functions the new sequence approximates better than the classical Bernstein polynomials.

Mathematics subject classification (2010): 41A36, 41A25, 41A40.

Keywords and phrases: Modified Bernstein-type operators, exponential functions, shape preserving properties, generalized convexity.

REFERENCES

[1] T. ACAR, A. ARAL, H. GONSKA, *On Szász-Mirakyan operators preserving e^{2ax}, $a > 0$*, Mediterr. J. Math. **14**: 6 (2017) 1–14.

[2] J. M. ALDAZ AND H. RENDER, *Optimalily of generalized Bernstein operators*, J. Approx. Theory **162** (2010) 1407–1416.

[3] J. M. ALDAZ, O. KOUNCHEV AND H. RENDER, *Bernstein operators for exponentials polynomials*, Constr. Approx. **29** (2009) 345–367.

[4] F. ALTOMARE AND M. CAMPITI, *Korovkin-type Approximation Theory and its Applications*, Walter de Gruyter, Berlin-New York, 1994.

[5] M. BESSENYEI AND Z. PÁLES, *Hadamard-type inequalities for generalized convex functions*, Math. Ineq. Appl. **6**, 3 (2003) 379–392.

[6] M. BIROU, *A note about some general King-type operators*, Ann. Tiberiu Popoviciu Semin. Funct. Equ. Approx. Convexity **12** (2014) 3–16.

[7] F. F. BONSALL, *The Characterization of Generalized Convex Functions*, Quart. J. Math. Oxford Ser. (2) **1** (1950) 100–111.

[8] D. CÁRDENAS-MORALES, P. GARRANCHO AND I. RAŞA, *Asymptotic Formulae via a Korovkin-Type Result*, Abstr. Appl. Anal. **2012**, Art. ID 217464, 12 pp.

[9] P. GARRANCHO AND D. CÁRDENAS-MORALES, *A converse of asymptotic formulae in simultaneous approximation*, Appl. Math. Comput. **217** (2010) 2676–2683.

[10] V. GUPTA AND G. TACHEV, *On Approximation Properties of Phillips Operators Preserving Exponential Functions*, Mediterr. J. Math. **14**: 177 (2017).

[11] S. KARLIN AND W. STUDDEN, *Tchebycheff Systems: with Applications inAnalysis and Statistics*, Interscience, New York, 1966.

[12] O. KOUNCHEV AND H. RENDER, *On the Bernstein operator of S. Morigi and M. Neamtu*, Result. Math. **53** (2009) 311–322.

[13] S. MORIGI AND M. NEAMTU, *Some results for a class of generalized polynomials*, Adv. Comput. Math. **12** (2000) 133–149.

[14] O. SHISHA AND B. MOND, *The degree of convergence of sequences of linear positive operators*, Proc. Nat. Acad. Sci. USA **60** (1968) 1196–1200.

[15] Z. ZIEGLER, *Linear approximation and generalized convexity*, J. Approx. Theory **1** (1968) 420–443.