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ON A NEW FAMILY OF BIVARIATE MEANS II

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Abstract. Lower and upper bounds as well as inequalities satisfied by members of a family of bivariate means, introduced recently by this author in [18], are established. In particular, optimal convex combinations bounds are obtained. Also, the Wilker and Huygens-type inequalities involving means under discussion are obtained.

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