A FAMILY OF WINDSCHITL TYPE APPROXIMATIONS FOR GAMMA FUNCTION

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Abstract. In this paper, we present a family of high accurate approximation formulas
\[
W_p(x) = \sqrt{2\pi x \left(\frac{x}{e}\right)^x (x \sinh \frac{1}{2})^{x/2} \exp\left(\frac{1}{1620x^5 x^2 + x^2 + \frac{33}{35}}\right)}
\]
for gamma function \(\Gamma(x+1)\) with parameter \(p \geq -\frac{33}{35}\), and prove the function
\[
x \mapsto \ln \Gamma(x+1) - \ln W_p(x)
\]
is strictly increasing and concave on \((0, \infty)\) if and only if \(p \geq \frac{158}{315}\). This yields some new
sharp approximations for gamma function.

Keywords and phrases: Gamma function, monotonicity, convexity, Windschitl type approximation.

REFERENCES