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COEFFICIENT PROBLEMS FOR UNIFIED STARLIKE AND CONVEX CLASSES OF m–FOLD SYMMETRIC BI–UNIVALENT FUNCTIONS

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Abstract. Let $\mathcal{T}_m$ denote the class of m-fold symmetric bi-univalent functions in the open unit disk. We obtain the coefficient bounds of $|a_{m+1}|$ and $|a_{2m+1}|$ for functions in a new general subclass $\mathcal{C}_m^{h,p}(\alpha)$ of $\mathcal{T}_m$, where h and p are in Carathéodary class of functions. We investigate the initial Taylor-Maclaurin coefficients estimate problems associated with $\mathcal{C}_m^{h,p}(\alpha)$ also. Our conclusion improves some earlier related results.

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