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ON THE INVERSE POWER INEQUALITY FOR THE BEREZIN NUMBER OF OPERATORS

MUBARIZ GARAYEV, SUNA SALTAN, DILARA GUNDOGDU

Abstract. The Berezin symbol $\widetilde{A}$ of operator A acting on the reproducing kernel Hilbert space $\mathscr{H} = \mathscr{H}(\Omega)$ over some set Ω is defined by

$$\widetilde{A}(\lambda) = \left\langle A\widehat{k}_{\mathscr{H},\lambda}, \widehat{k}_{\mathscr{H},\lambda} \right\rangle, \ \ \lambda \in \Omega,$$

where $\widehat{k}_{\mathscr{H},\lambda} = \frac{k_{\mathscr{H},\lambda}}{\|k_{\mathscr{H},\lambda}\|_{\mathscr{H}}}$ is the normalized reproducing kernel of $\mathscr{H}$. The Berezin number of operator A is the following number:

$$ber(A) := \sup\left\{ \left|\widetilde{A}(\lambda)\right| : \lambda \in \Omega \right\}.$$

Clearly, $ber(A) \leqslant w(A)$, where $w(A) = \sup\{|\langle Ax,x\rangle| : x \in \mathscr{H},\ \|x\|_{\mathscr{H}} = 1\}$ is the numerical radius of A. The power inequality for the numerical radius of Hilbert space operator A is the following:

$$w(A^n) \leqslant (w(A))^n, \ \ \forall n \geqslant 1.$$

Since $ber(A) \leqslant w(A)$, the following question naturally arises: is it true that $ber(A^n) \leqslant (ber(A))^n$ for any operator A and any integer $n > 1$?

Although we do not solve this question, in this paper, by using some Hardy type inequality, we prove the inverse power inequality for $ber(A)$ for positive operators on $\mathscr{H}(\Omega)$; namely, we prove that $(ber(A))^n \leqslant C(n,m)ber(A^n)$ for any positive operator A on $\mathscr{H}(\Omega)$, where $C(n,m) > 1$ is the constant depending only on n and its conjugate m, where $\frac{1}{n} + \frac{1}{m} = 1$.

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