GENERALIZED REVERSE CAUCHY INEQUALITY
AND APPLICATIONS TO OPERATOR MEANS

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Abstract. Let σ be an operator mean in the sense of Kubo-Ando and let ∇_α be a weighted arithmetic mean. If $\mathrm{Tr}(A\sigma B) \geqslant \mathrm{Tr}(A\nabla_\alpha B - \max\{\alpha, 1-\alpha\}|A-B|)$ holds for all positive semidefinite matrices A, B, then there exists $\beta \in [0,1]$ such that $\sigma = \nabla_\beta$.

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