CHOI–DAVIS–JENSEN'S INEQUALITY WITHOUT CONVEXITY

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Abstract. We give the Choi-Davis-Jensen type inequality without using convexity. Applying our main results, we also give new inequalities improving previous known results. In particular, we show some inequalities for relative operator entropies and quantum mechanical entropies.

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