

INEQUALITIES FOR THE WEIGHTED MEAN OF r -PREINVEK FUNCTIONS ON AN INVEK SET

DAH-YAN HWANG AND SILVESTRU SEVER DRAGOMIR

Abstract. In this paper, the inequalities for the weighted mean of weakly r -preinvex functions on an invex set are established. As applications, inequalities between the two-parameter mean of weakly r -preinvex functions and extended mean values are given.

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