ESTIMATES FOR PARAMETRIC MARCINKIEWICZ INTEGRALS ON MUSIELAK–ORLICZ HARDY SPACES

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Abstract. Let $\varphi : \mathbb{R}^n \times [0,\infty) \to [0,\infty)$ satisfy that $\varphi(x,\cdot)$, for any given $x \in \mathbb{R}^n$, is an Orlicz function and $\varphi(\cdot,t)$ is a Muckenhoupt A_∞ weight uniformly in $t \in (0,\infty)$. The Musielak-Orlicz Hardy space $H^\varphi(\mathbb{R}^n)$ generalizes both of the weighted Hardy space and the Orlicz Hardy space and hence has a wide generality. In this paper, the authors first prove the completeness of both of the Musielak-Orlicz space $L^\varphi(\mathbb{R}^n)$ and the weak Musielak-Orlicz space $WL^\varphi(\mathbb{R}^n)$. Then the authors obtain two boundedness criterions of operators on Musielak-Orlicz spaces. As applications, the authors establish the boundedness of parametric Marcinkiewicz integral μ_Ω^ρ from $H^\varphi(\mathbb{R}^n)$ to $L^\varphi(\mathbb{R}^n)$ (resp. $WL^\varphi(\mathbb{R}^n)$) under weaker smoothness condition (resp. some Lipschitz condition) assumed on Ω. These results are also new even when $\varphi(x,t) := \phi(t)$ for all $(x,t) \in \mathbb{R}^n \times [0,\infty)$, where ϕ is an Orlicz function.

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