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SHARP BOUNDS FOR SÁNDOR–YANG MEANS IN TERMS OF QUADRATIC MEAN

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Abstract. In the article, we find the best possible parameters α, β, λ, $\mu \in (1/2,1)$ such that the double inequalities

$$Q[\alpha a+(1-\alpha)b,\alpha b+(1-\alpha)a] < R_{QA}(a,b) < Q[\beta a+(1-\beta)b,\beta b+(1-\beta)a],$$

$$Q[\lambda a+(1-\lambda)b,\lambda b+(1-\lambda)a] < R_{AQ}(a,b) < Q[\mu a+(1-\mu)b,\mu b+(1-\mu)a]$$

hold for all $a,b>0$ with $a \neq b$, where $Q(a,b)=\sqrt{(a^2+b^2)/2}$ is the quadratic mean, and $R_{QA}(a,b)$ and $R_{AQ}(a,b)$ are two Sándor-Yang means.

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