

A PARAMETER-BASED OSTROWSKI TYPE INEQUALITY ON TIME SCALES WITH k POINTS FOR FUNCTIONS HAVING BOUNDED SECOND DERIVATIVES

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Abstract. In this paper, we introduce a parameter $\lambda \in [0, 1]$, and obtain a generalization of the Ostrowski type inequality on time scales for k points for functions whose second derivatives are bounded. Our results generalize some results in the literature and thereby, injects into the mathematical community some inequalities which we hope can be used to approximate the integral of a function in applied mathematics and/or mathematical physics. In addition, we apply our main theorem to the continuous, discrete, and quantum calculus to derive more inequalities in this direction.

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