

## COMPLETE MONOTONICITY PROPERTY FOR TWO FUNCTIONS RELATED TO THE $q$ -DIGAMMA FUNCTION

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*Abstract.* In this paper, the complete monotonicity property for two functions involving the  $q$ -digamma function are proven for all positive real  $q$  and exploited to establish some sharp inequalities for the  $q$ -gamma,  $q$ -digamma and  $q$ -polygamma functions. Comparisons between our results with previous results are provided.

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### REFERENCES

- [1] C. KRATTENTHALER, H. M. SRIVASTAVA, *Summations for basic hypergeometric series involving a  $q$ -analogue of the digamma function*, Computers Math. Applic., Vol. 32 (2) (1996) 73–91.
- [2] A. SALEM, *Some Properties and Expansions Associated with the  $q$ -Digamma Function*, Quaestiones mathematicae, Vol. 36 (1) (2013) 67–77.
- [3] M. E. H. ISMAIL AND M. E. MULDOON, *Inequalities and monotonicity properties for gamma and  $q$ -gamma functions*, in: R. V. M. Zahar (Ed.), Approximation and Computation, International Series of Numerical Mathematics, Vol. 119, Birkhauser, Boston, MA (1994) 309–323.
- [4] D. S. MOAK, *The  $q$ -analogue of Stirling's formula*, Rocky Mountain J. Math., Vol. 14 (1984) 403–412.
- [5] Z. CHENG, *Exact analytical study of ideal Bose atoms in a two-dimensional isotropic harmonic trap*, Journal of Statistical Mechanics: Theory and Experiment, Vol. 2017 (6), 063102 (2017).
- [6] Z. CHENG, *Bose-Einstein condensation of ideal photons in a one-dimensional barrel cavity*, Physical Review A, Vol. 93 (2) 023829 (2016).
- [7] Z. CHENG, *Exact spatial density of ideal Bose atoms in a one-dimensional harmonic trap*, Journal of Statistical Mechanics: Theory and Experiment, Vol. 2016 (5) 053102 (2016).
- [8] Z. CHENG, *Exact thermodynamic theory of an ideal boson gas in a one-dimensional harmonic trap*, Journal of Statistical Mechanics: Theory and Experiment, Vol. 2015 (11) P11003 (2015).
- [9] Z. CHENG, *Exact analytical study of ideal Bose atoms in a one-dimensional harmonic trap*, Journal of Statistical Mechanics: Theory and Experiment, Vol. 2015 (9), P09011 (2015).
- [10] N. BATIR, *Monotonicity properties of  $q$ -digamma and  $q$ -trigamma functions*, Journal of Approximation Theory, Vol. 192 (2015) 336–346.
- [11] N. BATIR,  *$q$ -Extensions of some estimates associated with the digamma function*, J. Approx. Theory **174** (2013) 54–64.
- [12] A. SALEM, *Sharp bounds for the  $q$ -gamma function in terms of the Lambert  $W$  function*, Ramanujan J., (2018) <https://doi.org/10.1007/s11139-018-0008-9> (In Press).
- [13] A. SALEM, F. ALZHRANI, *Improvements of bounds for the  $q$ -gamma and the  $q$ -polygamma functions*, Journal of Mathematical Inequalities, Vol. 11 (3) (2017) 873–883.
- [14] A. SALEM, *Completely monotonic functions related to the gamma and the  $q$ -gamma functions*, Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales – Serie A: Matemáticas, Vol. 111 (1) (2017) 271–280.
- [15] A. SALEM, *Some classes of completely monotonic functions related to  $q$ -gamma and  $q$ -digamma functions*, Mathematical Inequalities & Applications, Vol. 19 (3) (2016) 853–862.

- [16] A. SALEM, *A certain class of approximations for the  $q$ -digamma function*, Rocky Mountain Journal of Mathematics, Vol. 46 (5) (2016) 1665–1677.
- [17] A. SALEM, *Monotonic functions related to the  $q$ -gamma function*, Monatshefte für Mathematik, Vol. 179 (2) (2016) 281–292.
- [18] A. SALEM, *Some completely monotonic functions associated with the  $q$ -gamma and the  $q$ -polygamma functions*, Acta Mathematica Scientia, Vol. 35 (5) (2015) 1214–1224.
- [19] A. SALEM, *Completely monotonic functions related to the  $q$ -gamma and the  $q$ -trigamma functions*, Analysis & Applications, Vol. 13 (2) (2015) 125–134.
- [20] A. SALEM, *Complete monotonicity properties of functions involving  $q$ -gamma and  $q$ -digamma functions*, Mathematical Inequalities & Applications, Vol. 17 (3) (2014) 801–811.
- [21] A. SALEM, *Two classes of bounds for the  $q$ -gamma and the  $q$ -digamma functions in terms of the  $q$ -zeta functions*, Banach J. Math. Anal., Vol. 8 (1) (2014) 109–117.
- [22] A. SALEM, *An infinite class of completely monotonic functions involving the  $q$ -gamma function*, Journal of Mathematical Analysis and Applications, Vol. 406 (2) (2013) 392–399.
- [23] A. SALEM, *A completely monotonic function involving  $q$ -gamma and  $q$ -digamma functions*, Journal of Approximation Theory, Vol. 164 (7) (2012) 971–980.
- [24] P. GAO, *Some Monotonicity Properties of Gamma and  $q$ -Gamma Functions*, ISRN Mathematical Analysis, Vol. 2011 (2011) 1–15.
- [25] H. ALZER, A. Z. GRINSHPAN, *Inequalities for the gamma and  $q$ -gamma functions*, Journal of Approximation Theory, Vol. 144 (2007) 67–82.
- [26] A. Z. GRINSHPAN AND M. E. H. ISMAIL, *Completely monotonic functions involving the gamma and  $q$ -gamma functions*, Proc. Amer. Math. Soc, Vol. 134 (2006) 1153–1160.
- [27] N. ELEZOVIĆ, C. GIORDANO AND J. PEČARIĆ, *Convexity and  $q$ -gamma function*, Rendiconti del Circolo Matematico di Palermo, Vol. Serie II, Tomo XLVIII (1999) 285–298.
- [28] M. E. H. ISMAIL, L. LORCH AND M. E. MULDOON, *Completely monotonic functions associated with the gamma function and its  $q$ -analogues*, Journal of Mathematical Analysis and Applications, Vol. 116 (1986) 1–9.
- [29] M. ABRAMOWITZ, C. A. STEGUN, *Handbook of Mathematical functions with formulas, Graphs, Mathematical tables 7th printing*, Applied Mathematics Series, Vol. 55, National Bureau of Standards, Washington, DC, 1964.
- [30] N. BATIR, *Inequalities for the gamma function*, Arch. Math., Vol. 91 (2008) 554–563.