

SOME INEQUALITIES FOR THE L_p -CURVATURE IMAGES

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Abstract. Lutwak introduced the notion of L_p -curvature image and proved an inequality for volumes of convex body and its L_p -curvature image. In this article, based on the L_p -affine surface area and L_p -dual affine surface area, we establish the affine isoperimetric inequalities, cyclic inequalities and a monotonic inequality for L_p -curvature images.

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