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THE STABILITY OF AN ADDITIVE (ρ_1,ρ_2)–FUNCTIONAL INEQUALITY IN BANACH SPACES

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Abstract. In this paper, we introduce and solve the following additive (ρ_1,ρ_2)-functional inequality

$$\|f(x+y)-f(x)-f(y)\| \leqslant \|\rho_1(f(x+y)+f(x-y)-2f(x))\| + \left\|\rho_2\left(2f\left(\frac{x+y}{2}\right)-f(x)-f(y)\right)\right\|, \tag{1}$$

where ρ_1 and ρ_2 are fixed nonzero complex numbers with $\sqrt{2}|\rho_1|+|\rho_2|<1$.

Using the fixed point method and the direct method, we prove the Hyers-Ulam stability of the additive (ρ_1,ρ_2)-functional inequality (1) in complex Banach spaces.

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