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THE ESTIMATE OF THE DIFFERENCE OF INITIAL SUCCESSIVE COEFFICIENTS OF UNIVALENT FUNCTIONS

ZHIGANG PENG AND MILUTIN OBRADOVIĆ

Abstract. Let $\mathscr{A}$ denote the family of all functions that are analytic in the unit disk $\mathbb{D} := \{z : |z| < 1\}$ and satisfy $f(0) = 0 = f'(0) - 1$. Let S be the set of all functions $f \in \mathscr{A}$ that are univalent in $\mathbb{D}$. In this paper the sharp upper bounds of $|a_3 - a_2|$ and $|a_4 - a_3|$ for the functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ being in several subclasses of S are presented.

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