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LAWS OF LARGE NUMBERS WITH INFINITE MEAN

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Abstract. In this paper, we study the weak law and strong law of large numbers based on $\tilde{\rho}$-mixing random variables with infinite mean. If the random variables satisfy the Pareto type distributions, then some weak laws of large numbers are presented. If the random variables satisfy the two tailed Pareto distribution and asymmetrical Cauchy distribution, the strong laws of large numbers are also obtained. Furthermore, we do some simulations for the laws of large numbers for two tailed Pareto distribution and asymmetrical Cauchy distribution.

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