

REFINEMENTS OF THE HADAMARD AND CAUCHY–SCHWARZ INEQUALITIES WITH TWO INEQUALITIES OF THE PRINCIPAL ANGLES

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Abstract. By discussing two volume formulae for the parallelotope, some refinements of the Hadamard and Cauchy-Schwarz inequalities are given and a class of principal inequalities related a parallelotope is established. This class of principal inequalities have a close relation to the Hadamard and Fischer determinant inequalities. By using the interlacing property, a principal inequality related to two subspaces is given which has a close relation to the Kotljanskii determinant inequality. Analysis indicates that these two principal inequalities can be extended to two class of principal inequalities easily.

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