Journal of Mathematical Inequalities
Volume 13, Number 2 (2019), 479–493
doi:10.7153/jmi-2019-13-32

THE HARMONIC INDEX OF TWO–TREES AND QUASI–TREE GRAPHS

XIAOLING SUN, YUBIN GAO AND JIANWEI DU

Abstract. The harmonic index of a graph G is defined as the sum of the weights $\frac{2}{d(u)+d(v)}$ of all edges uv of G, where $d(u)$ denotes the degree of the vertex u in G. A graph G is called quasi-tree, if there exists $u \in V(G)$ such that $G-u$ is a tree. The graphs called two-trees are defined by recursion. The smallest two-tree is the complete graph on two vertices. A two-tree on $n+1$ vertices (where $n \geqslant 2$) is obtained by adding a new vertex adjacent to the two end vertices of one edge in a two-tree on n vertices. In this work, the sharp lower and upper bounds on the harmonic index of quasi-tree graphs are presented. Furthermore, the lower bound on the harmonic index of two-trees is presented, and the two-trees with the minimum and the second minimum harmonic index, respectively, are determined.

Mathematics subject classification (2010): 05C07, 05C15.
Keywords and phrases: Harmonic index, two-tree, quasi-tree graph, cycle.

REFERENCES

[1] L. ZHONG, *The harmonic index for graphs*, Appl. Math. Lett., **25**, (2012), 561–566.
[2] S. FAJTLOWICZ, *On conjectures of Graffiti-II*, Congr. Numer., **60**, (1987), 187–197.
[3] R. WU, Z. TANG AND H. DENG, *A lower bound for the harmonic index of a graph with minimum degree at least two*, Filomat, **27**, (2013), 51–55.
[4] L. ZHONG, *The harmonic index on unicyclic graphs*, Ars Combin., **104**, (2012), 261–269.
[5] L. ZHONG AND K. XU, *The harmonic index for bicyclic graphs*, Utilitas Math., **90**, (2013), 23–32.
[6] S. WANG, B. ZHOU AND N. TRINAJSTIĆ, *On the sum-connectivity index*, Filomat, **25**, 3 (2011), 29–42.
[7] X. XU, *Relationships between harmonic index and other topological indices*, Applied Mathematical Sciences, **6**, (2012), 2013–2018.
[8] A. LLIĆ, *Note on the harmonic index of a graph*, Arxiv: 1204.3313v1 [math.CO], (2012).
[9] C. DELORME, O. FAVARON AND D. RAUTENBACH, *On the Randić index*, Discrete Math., **257**, (2002), 29–38.
[10] H. DENG, Z. TANG AND J. ZHANG, *On a conjecture of Randić index and graph radius*, Filomat, **29**, (2011), 1369–1375.
[11] J. GAO AND M. LU, *On the Randić index of unicyclic graphs*, MATCH Commun. Math. Comput. Chem., **53**, (2005), 377–384.
[12] S. LIU AND J. LI, *Some properties on the harmonic index of molecular trees*, ISRN Applied Mathematics, **2014**, (2014), 1–17.
[13] R. WU, Z. TANG AND H. DENG, *On the harmonic index and the girth of a graph*, Utilitas Math., **91**, (2013), 65–69.
[14] R. TODESCHINI AND V. CONSONNI, *Handbook of Molecular Descriptors*, Wiley-VCH, Weinheim, 2000.
[15] J. A. BONDY AND U. S. R. MURTY, *Graph theory with applications*, Elsevier, New York, 1976.
[16] L. ZHONG AND Q. CUI, *The harmonic index for unicyclic graphs with given girth*, Filomat, **29**, 4 (2015), 673–686.
[17] J. LI, J. LV AND Y. LIU, *The harmonic index of some graphs*, Bull. Malays. Math. Sci. Soc., **39**, (2016), 331–340.

Paper JMI-13-32

[18] H. DENG, S. BALACHANDRAN, S. K. AYYASWAMY AND Y. B. VENKATAKRISHNAN, *The harmonic indices of polyomino chains*, Natl. Acad. Sci. Lett., **37**, 5 (2014), 451–455.
[19] L. ZHONG, *Note on a relation between the harmonic index and the average distances of trees*, Utilitas Math., **96**, (2015), 277–283.
[20] L. ZHONG, *The harmonic index for unicyclic and bicyclic graphs with given matching number*, Miskolc Math. Notes, **16**, 1 (2015), 587–605.
[21] Y. ZHU AND R. CHANG, *Maximum harmonic indices of trees and unicyclic graphs with given number of pendant vertices and diameter*, Utilitas Math., **93**, (2014), 365–374.
[22] Z. ZHANG, B. WU AND Y. LIN, *Counting spanning trees in a small-world Farey graph*, Phys. A, **391**, (2012), 3342–3349.
[23] Z. ZHANG, H. LIU, B. WU AND T. ZHOU, *Spanning trees in a fractal scale-free lattice*, Phys. Rev. E, **83**, (2011), 129–148.
[24] Z. ZHANG, H. LIU, B. WU AND S. ZHOU, *Enumeration of spanning trees in a pseudofractal scale-free web*, Eur. Phys. Lett., **90**, (2010), 1632–1652.
[25] Z. ZHANG, L. RONG AND C. GUO, *A deterministic small-world network created by edge iterations*, Phys. A, **363**, (2006), 567–572.