

GENERALIZATIONS AND REFINEMENTS OF STEČKIN-TYPE INEQUALITY FOR TANGENT AND SECANT FUNCTIONS

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Abstract. In this paper, we generalize and refine Stečkin-type inequality for the tangent function. We develop an inequality of Chen and Sándor for the secant function to produce a general form. We also present some refinements of the inequality for the secant function.

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REFERENCES

- [1] M. ABRAMOWITZ AND I. A. STEGUN (Editors), *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Dover, New York, 1965.
- [2] H. ALZER, *Sharp bounds for the Bernoulli numbers*, Arch. Math. **74** (2000), 207–211.
- [3] G. D. ANDERSON, S. - L. QIU, M. K. VAMANAMURTHY AND M. VUORINEN, *Generalized elliptic integral and modular equations*, Pacific J. Math. **192** (2000), 1–37.
- [4] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, New York, 1997.
- [5] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Monotonicity of Some Functions in Calculus*, available at <http://www.math.auckland.ac.nz/Research/Reports/Series/538.pdf>.
- [6] D. S. MITRINOVIĆ, *Analytic Inequalities*, Springer, New York, 1970.
- [7] B. BANJAC, M. MAKRAGIĆ AND B. MALEŠEVIĆ, *Some notes on a method for proving inequalities by computer*, Results. Math. **69**, 1 (2016), 161–176.
- [8] M. BECKER AND E. L. STARK, *On a hierarchy of quolynomial inequalities for $\tan x$* , Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat. Fiz. No. 602-633 (1978), 133–138.
- [9] C. - P. CHEN AND W. - S. CHEUNG, *Sharp Cusa and Becker–Stark inequalities*, J. Inequal. Appl. **2011** (2011) 136, <http://www.journalofinequalitiesandapplications.com/content/2011/1/136>.
- [10] C. - P. CHEN AND N. ELEZOVIĆ, *Sharp Redheffer-type and Becker–Stark-type inequalities with an application*, Math. Inequal. Appl. **21**, 4 (2018), 1059–1078.
- [11] C. - P. CHEN AND R. B. PARIS, *Series representations of the remainders in the expansions for certain trigonometric functions and some related inequalities, I*, Math. Inequal. Appl. **20**, 4 (2017), 1003–1016.
- [12] C. - P. CHEN AND J. SÁNDOR, *Sharp inequalities for trigonometric and hyperbolic functions*, J. Math. Inequal. **9**, 1 (2015), 203–217.
- [13] L. DEBNATH, C. MORTICI AND L. ZHU, *Refinements of Jordan–Stečkin and Becker–Stark inequalities*, Results Math. **67** (2015), 207–215.
- [14] H. - F. GE, *New Sharp Bounds for the Bernoulli Numbers and Refinement of Becker–Stark Inequalities*, J. Inequal. Appl. **2012**, Article ID 137507, 7 pages.
- [15] I. S. GRADSHTEYN, I. M. RYZHIK, *Table of integrals, series, and products*, translated from the Russian, sixth edition, translation edited and with a preface by Alan Jeffrey and Daniel Zwillinger, Academic Press, Inc., San Diego, CA, 2000.

- [16] Y. NISHIZAWA, *Sharp Becker–Stark’s type inequalities with power exponential functions*, J. Inequal. Appl. **2015** (2015) 402, <http://rd.springer.com/article/10.1186/s13660-015-0932-9/fulltext.html>.
- [17] Y. NISHIZAWA, *Sharp exponential approximate inequalities for trigonometric functions*, Results Math. **71** (2017), 609–621.
- [18] S. B. STEČKIN, *Some remarks on trigonometric polynomials*, Uspekhi Matematicheskikh Nauk, vol. 10, no. 1 (63), (1955) 159–166 (in Russian).
- [19] Z. - J. SUN AND L. ZHU, *Simple proofs of the Cusa–Huygens–type and Becker–Stark–type inequalities*, J. Math. Inequal. **7** (2013), 563–567.
- [20] L. ZHU, *Sharp Becker–Stark–type inequalities for Bessel functions*, J. Inequal. Appl. **2010**, Article ID 838740, 4 pages.
- [21] L. ZHU, *A refinement of the Becker–Stark inequalities*, Math. Notes **93** (2013), 421–425.
- [22] L. ZHU AND J. K. HUA, *Sharpening the Becker–Stark inequalities*, J. Inequal. Appl. **2010**, Article ID 931275, 4 pages.