

## EQUALITY CONDITIONS OF REVERSE SCHWARZ AND GRÜSS INEQUALITIES IN INNER PRODUCTSPACES

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*Abstract.* The cases of equality for reverse Schwarz inequalities and Grüss type inequalities are detailed studied. Necessary and sufficient conditions for them are given. Moreover, we introduce an unification of two reverse Schwarz inequalities obtained by S.S. Dragomir [S.S. Dragomir, *Advances in Inequalities of the Schwarz, Grüss and Bessel Type in Inner Product Spaces*, Chap. I, th. 1, 6. Nova Science Publishers, New York 2005] and, as an application, we gain more general versions of Grüss inequalities.

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