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SOME INEQUALITIES FOR GENERALIZED BELL–TOUCHARD POLYNOMIALS

HAI-RONG YAN, QIAO-LING ZHANG AND AI-MIN XU

Abstract. A unified generalization for the Bell-Touchard polynomials of order k and the r-Bell polynomials is established. It is shown that the generating function of the generalized Bell-Touchard polynomials is logarithmically absolutely monotonic. Applying this result we obtain some inequalities for the generalized Bell-Touchard polynomials. In particular, we obtain the logarithmic convexity of the generalized Bell-Touchard polynomials.

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