

## $L_p$ -DUAL AFFINE SURFACE AREAS FOR THE GENERAL $L_p$ -CENTROID BODIES

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*Abstract.* Lutwak and Zhang proposed the concept of  $L_p$ -centroid bodies. Further, Haberl and Schuster extended this notion to the general  $L_p$ -centroid bodies. In this paper, associated with the  $L_p$ -dual affine surface areas, we give the extremum values of polar for the general  $L_p$ -centroid bodies. Moreover, the  $L_p$ -dual affine surface area forms of the Brunn-Minkowski type inequality and a monotone inequality are established.

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