A NEW PROOF OF THE ORLICZ–LORENTZ
BUSEMANN–PETTY CENTROID INEQUALITY

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Abstract. The Orlicz-Lorentz Busemann-Petty centroid inequality was recently established by Nguyen [16]. In this paper, using shadow systems, a new proof of the Orlicz-Lorentz Busemann-Petty centroid inequality is given.

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