

LYAPUNOV-TYPE INEQUALITY FOR THE HADAMARD FRACTIONAL BOUNDARY VALUE PROBLEM ON A GENERAL INTERVAL $[a, b]$

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Abstract. In this paper, using two different methods, we studied an open problem and obtained several results for Lyapunov-type and Hartman-Wintner-type inequalities for a Hadamard fractional differential equation on a general interval $[a, b]$, $(1 \leq a < b)$ with the boundary value conditions.

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