A REFINEMENT OF THE KOOI'S INEQUALITY,
MITTENPUNKT AND APPLICATIONS

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Abstract. Kooi's inequality is important in the theory of triangle inequalities, giving bound on the semiperimeter in terms of circumradius and inradius. We give one refinement, then relate the inequality to the Mittenpunk, and finally apply it to improve several known inequalities.

Mathematics subject classification (2010): 51M04, 51M16.

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