

ADDITIVE s -FUNCTIONAL INEQUALITIES AND PARTIAL MULTIPLIERS IN BANACH ALGEBRAS

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Abstract. In this paper, we solve the additive s -functional inequalities

$$\|f(x+y-z) - f(x) - f(y) + f(z)\| \leq \|s(f(x-y) + f(y-z) - f(x-z))\|, \quad (0.1)$$

where s is a fixed nonzero complex number with $|s| < 1$, and

$$\|f(x-y) + f(y-z) - f(x-z)\| \leq \|s(f(x+y-z) - f(x) - f(y) + f(z))\|, \quad (0.2)$$

where s is a fixed nonzero complex number with $|s| < 1$.

Furthermore, we prove the Hyers-Ulam stability of the additive s -functional inequalities (0.1) and (0.2) in complex Banach spaces. This is applied to investigate partial multipliers in Banach $*$ -algebras and unital C^* -algebras, associated with the additive s -functional inequalities (0.1) and (0.2).

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