FURTHER INEQUALITIES FOR THE NUMERICAL
RADIUS OF HILBERT SPACE OPERATORS

Sara Tafazoli, Hamid Reza Moradi, Shigeru Furuichi and Panackal Harikrishnan

Abstract. In this article, we present some new inequalities for numerical radius of Hilbert space operators via convex functions. Our results generalize and improve earlier results by El-Haddad and Kittaneh. Among several results, we show that if $A \in \mathbb{B}(\mathscr{H})$ and $r \geqslant 2$, then

$$w^r(A) \leqslant \|A\|^r - \inf_{\|x\|=1} \left\| \, |A| - w(A)|^{\frac{r}{2}} x \right\|^2$$

where $w(\cdot)$ and $\|\cdot\|$ denote the numerical radius and usual operator norm, respectively.

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