Journal of Mathematical Inequalities
Volume 13, Number 4 (2019), 969–981
doi:10.7153/jmi-2019-13-69

AN INEQUALITY FOR DISTANCES AMONG n POINTS AND DISTANCE PRESERVING MAPPINGS

SOON-MO JUNG AND DOYUN NAM

Abstract. Using familiar properties of norm and inner product, we will prove a new inequality concerning distances between each pair of n points in an inner product space, where n is an integer larger than 3. Moreover, we investigate the Aleksandrov-Rassias problem by proving that if the distance 1 is contractive and the golden ratio is extensive by a mapping f, then f is a linear isometry up to translation.

Mathematics subject classification (2010): 51M16, 51K99, 51K05, 46C99.
Keywords and phrases: Inequalities, Aleksandrov-Rassias problem, distance, linear isometry, point.

REFERENCES

[1] A. D. ALEKSANDROV, *Mapping of families of sets*, Soviet Math. Dokl. **11** (1970), 116–120.
[2] F. S. BECKMAN AND D. A. QUARLES, *On isometries of Euclidean spaces*, Proc. Amer. Math. Soc. **4** (1953), 810–815.
[3] W. BENZ, *Isometrien in normierten Räumen*, Aequationes Math. **29** (1985), 204–209.
[4] W. BENZ AND H. BERENS, *A contribution to a theorem of Ulam and Mazur*, Aequationes Math. **34** (1987), 61–63.
[5] S.-M. JUNG, *Inequalities for distances between points and distance preserving mappings*, Nonlinear Anal. **62** (2005), no. 4, 675–681.
[6] S.-M. JUNG AND K.-S. LEE, *An inequality for distances between $2n$ points and the Aleksandrov-Rassias problem*, J. Math. Anal. Appl. **324** (2006), no. 2, 1363–1369.
[7] S.-M. JUNG AND D. NAM, *An inequality for distances among five points and distance preserving mappings*, J. Math. Inequal. **12** (2018), no. 4, 1189–1199.
[8] TH. M. RASSIAS, *Is a distance one preserving mapping between metric spaces always an isometry?* Amer. Math. Monthly **90** (1983), 200.
[9] TH. M. RASSIAS, *Mappings that preserve unit distance*, Indian J. Math. **32** (1990), 275–278.
[10] TH. M. RASSIAS, *Properties of isometries and approximate isometries*, in Recent Progress in Inequalities (ed. G. V. Milovanovic), pp. 341–379, Kluwer, 1998.
[11] S. XIANG, *Mappings of conservative distances and the Mazur-Ulam theorem*, J. Math. Anal. Appl. **254** (2001), 262–274.