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A PROOF OF HILBERT'S THEOREM ON TERNARY QUARTIC FORMS

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Abstract. Hilbert's theorem states that every positive semi-definite real ternary quartic form can be written as a sum of squares of quadratic forms. In this paper, a constructive proof based on the method called ladder technique is presented. A practical example is proposed to illustrate that this method can be used to prove some hard inequalities on ternary quartic forms.

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