

SHARP L^p HARDY TYPE AND UNCERTAINTY PRINCIPLE INEQUALITIES ON THE SPHERE

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Abstract. This paper studies L^p -version of the Hardy type inequalities on the geodesic sphere of constant sectional curvature and establishes that the corresponding constant is sharp. Furthermore, the inequalities obtained are used to derive an uncertainty principle inequality and another inequality involving the first nonzero eigenvalue of the p -Laplacian on the sphere.

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