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SOME IMPROVEMENTS ON THE L_p INEQUALITIES FOR DIFFUSION PROCESSES

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Abstract. In this paper, we give some improvements on the $L_p(0 < p < \alpha)$ inequalities for diffusion processes. We obtain smaller constants in the L_p inequalities and derive that the growth rates of the constants, as $p \to 0^+$, grows like $O\left(\frac{1}{p^\alpha}\right)$, instead of the exponential of $\frac{1}{p}$. Finally, we apply the improved inequalities to the Ornstein-Uhlenbeck processes, Bessel processes and reflected Brownian motion with drift and get better constants.

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