

REFINEMENTS OF CAUCHY–SCHWARZ NORM INEQUALITY

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Abstract. In this paper, we utilize the convexity of the function $f(v) = \| |A^v X B^{1-v}|^r \| \cdot \| |A^{1-v} X B^v|^r \|$ and the Hermite–Hadamard inequality to obtain a family of new refinements of Cauchy–Schwarz norm inequality for operators, which extends the related results.

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