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BRAUER–TYPE UPPER BOUNDS FOR Z–SPECTRAL RADIUS OF WEAKLY SYMMETRIC NONNEGATIVE TENSORS

GANG WANG, YANAN WANG AND YUAN ZHANG

Abstract. In this paper, we establish four Brauer-type upper bounds and a lower bound for Z-spectral radius of weakly symmetric nonnegative tensors without irreducible assumption. These bounds are shown to be sharper than the existing bounds via running examples. As an application, an upper bound on the largest Z-eigenvalues of the adjacency tensors for uniform hypergraphs is provided.

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