COMPLETE REFINEMENTS OF THE BEREZIN NUMBER INEQUALITIES

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Abstract. In this paper, several refinements of the Berezin number inequalities are obtained. We generalize inequalities involving powers of the Berezin number for product of two operators acting on a reproducing kernel Hilbert space $\mathcal{H} = \mathcal{H}(\Omega)$ and also improve them. Among other inequalities, it is shown that if $A, B \in B(\mathcal{H})$ such that $|A|B = B^*|A|$, $f$ and $g$ are nonnegative continuous functions on $[0, \infty)$ satisfying $f(t)g(t) = t^r$ ($r \geq 0$), then

$$\text{ber}^p(AB) \leq r^p(B) \times$$

$$\left( \text{ber} \left( \frac{1}{\alpha} f^{\alpha p}(|A|) + \frac{1}{\beta} g^{\beta p}(|A^*|) \right) - r_0 \left( \langle f^2(|A|) \hat{k}_k, \hat{k}_\lambda \rangle^{\alpha p/4} - \langle g^2(|A^*|) \hat{k}_k, \hat{k}_\lambda \rangle^{\beta p/4} \right)^2 \right)$$

for every $p \geq 1, \alpha \geq \beta > 1$ with $\frac{1}{\alpha} + \frac{1}{\beta} = 1$, $\beta p \geq 2$ and $r_0 = \min\{\frac{1}{\alpha}, \frac{1}{\beta}\}$.


Keywords and phrases: Berezin number; Berezin symbol; Heinz means.

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