

## LYAPUNOV-TYPE INEQUALITIES FOR CERTAIN HIGHER-ORDER HALF-LINEAR DIFFERENTIAL EQUATIONS

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*Abstract.* In this paper, we will establish some new Lyapunov-type inequalities for some higher-order half-linear differential equations with anti-periodic boundary conditions. Our results not only improve the results in [9] for some cases, but also extend the results of [11].

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