SHARPNESS AND GENERALIZATION OF JORDAN, BECKER–STARK
AND PAPENFUSS INEQUALITIES WITH AN APPLICATION

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Abstract. In this paper, we present an identity related to Jordan’s inequality. More precisely, we provide a formula for determining the coefficients $b_n \equiv b_n(\theta)$ such that

$$\frac{\sin x}{x} = \sum_{n=0}^{\infty} b_n (\pi \theta - (2x)^\theta)^n,$$

where $\theta \geq 2$ is a given real number. We present a generalization of Jordan’s inequality. As an application, we improve the well-known Yang Le inequality. We establish sharp bounds for $(\tan x/x)^{(n)}$ for $n = 0$ and $n = 1$. Further, an interesting open problem and a conjecture regarding our present concern are posed.


Keywords and phrases: Jordan’s inequality; Becker-Stark inequality; Papenfuss inequality; Yang Le inequality; Bell polynomials of the second kind; Faà di Bruno formula.

REFERENCES


