3–VARIABLE DOUBLE ρ–FUNCTIONAL INEQUALITIES OF DRYGAS

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Abstract. Drygas introduced the functional equation $f(x+y)+f(x-y)=2f(x)+f(y)+f(-y)$ in quasi-inner product spaces. In this paper, we introduce and solve 3-variable double ρ-functional inequalities associated to the functional equation $f(x+y+z)+f(x+y-z)=2f(x)+2f(y)+f(z)+f(-z)$. Moreover, we prove the Hyers-Ulam stability of the 3-variable double ρ-functional inequalities in complex Banach spaces.

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