MONOTONICITY PROPERTIES ON k–DIGAMMA
FUNCTION AND ITS RELATED INEQUALITIES

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Abstract. In this work, we give some monotonicity properties of k-analogues of digamma and polygamma functions and then we obtain some inequalities related to these functions. At last, we give harmonic mean inequality for k-digamma function for all positive real values of k and x.

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