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A NEW FAMILY OF WEIGHTED OPERATOR MEANS INCLUDING THE WEIGHTED HERON, LOGARITHMIC AND HEINZ MEANS

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Abstract. The weighted power and Heron means are well known as generalizations of the weighted arithmetic, geometric and harmonic ones, and also some researchers investigate weighted means except them. Recently, Pal, Singh, Moslehian and Aujla introduced the weighted logarithmic mean of two positive numbers or operators.

In this paper, we propose the notion of a transpose symmetric path of weighted $\mathfrak{M}$-means for a symmetric operator mean $\mathfrak{M}$, and we introduce a new family of operator means including the weighted logarithmic mean by Pal et al.. This family newly produces the weighted Heinz mean. Moreover we obtain some relations among the weighted Heron, logarithmic and Heinz means.

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