GENERALIZATIONS OF CYCLIC REFINEMENTS OF JENSEN'S INEQUALITY BY LIDSTONE'S POLYNOMIAL WITH APPLICATIONS IN INFORMATION THEORY

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Abstract. Jensen's inequality plays pivotal role in attaining divergence between probability distributions. Shannon, Relative and Zipf-Mandelbrot entropies have ample applications in many applied sciences, especially in information theory, biology, economics, etc. In the present paper, we have obtained new generalizations of cyclic refinements of Jensen's inequality using different new Green functions by employing Lidstone's polynomial. As an application of our obtained results we have given new entropic bounds. Also, we have established the connections between Shannon and Relative entropy with Zipf-Mandelbrot entropy.

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