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ON APPROXIMATION OF FUNCTION IN GENERALIZED ZYGMUND CLASS USING $C^{\eta}T$ OPERATOR

H. K. NIGAM AND MD. HADISH

Abstract. In the present work, we, for the first time, study the error estimates of a function h $(2\pi$-periodic) in generalized Zygmund class Z_r^u $(r \geqslant 1)$ by Cesàro-Matrix $(C^{\eta}T)$ product means of its Fourier series (F. S.). The results obtained in the paper provide the best approximation of the function h in Z_q^u $(q \geqslant 1)$ class. Our Theorem 2.1 generalizes eight previously known results. Thus, the results of Singh and Srivastava [29], Lal and Kushwaha [25], Lal [23], Nigam and Sharma [10], Nigam [8], Lal [22] and Uğur Değer [28] become the particular cases of our Theorem 2.1. Several useful results in the form of corollaries are also achieved from the main theorems.

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