ULAM STABILITY OF AN ADDITIVE–QUADRATIC
FUNCTIONAL EQUATION IN BANACH SPACES

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Abstract. Using the fixed point method and the direct method, we prove the Hyers-Ulam stability of the following additive-quadratic functional equation

$$f(x+y,z+w) + f(x-y,z-w) - 2f(x,z) - 2f(x,w) = 0. \qquad (0.1)$$

REFERENCES

[1] T. AOKI, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan **2** (1950), 64–66.

[2] L. CĂDARIU, V. RADU, *Fixed points and the stability of Jensen's functional equation*, J. Inequal. Pure Appl. Math. **4**, no. 1, Art. ID 4 (2003).

[3] L. CĂDARIU, V. RADU, *On the stability of the Cauchy functional equation: a fixed point approach*, Grazer Math. Ber. **346** (2004), 43–52.

[4] L. CĂDARIU, V. RADU, *Fixed point methods for the generalized stability of functional equations in a single variable*, Fixed Point Theory Appl. **2008**, Art. ID 749392 (2008).

[5] J. DIAZ, B. MARGOLIS, *A fixed point theorem of the alternative for contractions on a generalized complete metric space*, Bull. Am. Math. Soc. **74** (1968), 305–309.

[6] Y. DING, T.-Z. XU, *Approximate solution of generalized inhomogeneous radical quadratic functional equations in 2-Banach spaces*, J. Inequal. Appl. **2019**, 2019:31, 13 pp.

[7] A. EBADIAN, S. ZOLFAGFHARI, S. OSTADBASHI, C. PARK, *Approximation on the reciprocal functional equation in several variables in matrix non-Archimedean random normed spaces*, Adv. Difference Equ. **2015**, 2015:314, 13 pp.

[8] I. EL-FASSI, *Generalized hyperstability of a Drygas functional equation on a restricted domain using Brzdek's fixed point theorem*, J. Fixed Point Theory Appl. **19** (2017), 2529–2540.

[9] I. EL-FASSI, *Solution and approximation of radical quintic functional equation related to quintic mapping in quasi-β -Banach spaces*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. **113** (2019), no. 2, 675–687.

[10] M. ESHAGHI GORDJI, N. GHOBADIPOUR, *Stability of (α, β, γ)-derivations on Lie C^*-algebras*, Int. J. Geom. Meth. Mod. Phys. **7** (2010), 1097–1102.

[11] W. FECHNER, *Stability of a functional inequalities associated with the Jordan-von Neumann functional equation*, Aequationes Math. **71** (2006), 149–161.

[12] P. GĂVRUTA, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl. **184** (1994), 431–436.

[13] A. GILÁNYI, *Eine zur Parallelogrammgleichung äquivalente Ungleichung*, Aequationes Math. **62** (2001), 303–309.

[14] A. GILÁNYI, *On a problem by K. Nikodem*, Math. Inequal. Appl. **5** (2002), 707–710.

[15] D.H. HYERS, *On the stability of the linear functional equation*, Proc. Nat. Acad. Sci. U.S.A. **27** (1941), 222–224.

[16] G. ISAC, TH. M. RASSIAS, *Stability of ψ -additive mappings: Applications to nonlinear analysis*, Int. J. Math. Math. Sci. **19** (1996), 219–228.

[17] D. MIHEŢ, V. RADU, *On the stability of the additive Cauchy functional equation in random normed spaces*, J. Math. Anal. Appl. **343** (2008), 567–572.

[18] C. PARK, *Additive ρ -functional inequalities and equations*, J. Math. Inequal. **9** (2015), 17–26.

[19] C. PARK, *Additive ρ -functional inequalities in non-Archimedean normed spaces*, J. Math. Inequal. **9** (2015), 397–407.

[20] C. PARK, *Fixed point method for set-valued functional equations*, J. Fixed Point Theory Appl. **19** (2017), 2297–2308.

[21] V. RADU, *The fixed point alternative and the stability of functional equations*, Fixed Point Theory **4** (2003), 91–96.

[22] TH. M. RASSIAS, *On the stability of the linear mapping in Banach spaces*, Proc. Am. Math. Soc. **72** (1978), 297–300.

[23] J. RÄTZ, *On inequalities associated with the Jordan-von Neumann functional equation*, Aequationes Math. **66** (2003), 191–200.

[24] L. SZÉKELYHIDI, *Superstability of functional equations related to spherical functions*, Open Math. **15** (2017), no. 1, 427–432.

[25] S. M. ULAM, *A Collection of the Mathematical Problems*, Interscience Publ. New York, 1960.

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