SOME INEQUALITIES FOR RECIPROCALLY (s,m)—CONVEX IN THE SECOND SENSE FUNCTIONS AND APPLICATIONS TO SPECIAL MEANS

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Abstract. We present the notion of reciprocally (s,m)–convex functions and present some examples and properties of them. We derive some inequalities for this new class of functions, specifically these inequalities are: Hermite–Hadamard and Fejér. In addition, we present some applications of our results to special media of positive real numbers.

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