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A GENERALIZATION OF THE BOUNDEDNESS OF CERTAIN INTEGRAL OPERATORS IN VARIABLE LEBESGUE SPACES

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Abstract. Let $n \in \mathbb{N}$. Let $A_1, ...A_m$ be $n \times n$ invertible matrices. Let $0 \leqslant \alpha < n$ and $0 < \alpha_i < n$ such that $\alpha_1 + ... + \alpha_m = n - \alpha$. We define

$$T_\alpha f(x) = \int \frac{1}{|x - A_1 y|^{\alpha_1} ... |x - A_m y|^{\alpha_m}} f(y) dy.$$

In [8] we obtained the boundedness of this operator from $L^{p(.)}(\mathbb{R}^n)$ into $L^{q(.)}(\mathbb{R}^n)$ for $\frac{1}{q(.)} = \frac{1}{p(.)} - \frac{\alpha}{n}$, in the case that A_i is a power of certain fixed matrix A and for exponent functions p satisfying log-Hölder conditions and $p(Ay) = p(y)$, $y \in \mathbb{R}^n$. We will show now that the hypothesis on p, in certain cases, is necessary for the boundedness of T_α and we also prove the result for more general matrices A_i.

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