

REVERSING BELLMAN OPERATOR INEQUALITY

MOHAMMAD SABABHEH, HAMID REZA MORADI AND SHIGERU FURUICHI

Abstract. The main aim of the present paper is to obtain several reverses of the operator Bellman inequality. To this end, we employ Mond-Pečarić method to achieve a general inequality treating the arithmetic mean and unital positive linear maps. In particular, we show that, for certain scalars α, β ,

$$\alpha(\Phi(I - A\nabla_v B))^{1/p} + \beta I \leq \Phi\left((I - A)^{1/p} \nabla_v (I - B)^{1/p}\right)$$

for the positive operators A, B , the normalized positive linear map Φ and $p > 1$. As a consequence, we get multiplicative and additive reverses of operator Bellman inequality. Further, we show some inequalities involving concave and convex functions. In the end, we present a simple proof of the scalar Bellman inequality and its reverses.

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