A TRUDINGER–MOSER TYPE INEQUALITY AND ITS EXTREMAL FUNCTIONS IN DIMENSION TWO

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Abstract. Let $\Omega$ be a smooth bounded domain in $\mathbb{R}^2$, $W^{1,2}_0(\Omega)$ be the usual Sobolev space and $\lambda(\Omega)$ be the first eigenvalue of the Laplace-Beltrami operator, say

$$\lambda(\Omega) = \inf_{u \in W^{1,2}_0(\Omega), \int_{\Omega} u^2 dx = 1} \int_{\Omega} |\nabla u|^2 dx.$$

Using blow-up analysis, we prove that for real numbers $\alpha < \lambda(\Omega)$ and $\beta < 4\pi$, the supremum

$$\sup_{u \in W^{1,2}_0(\Omega), \int_{\Omega} |\nabla u|^2 dx - \alpha \int_{\Omega} u^2 dx \leq 1} \int_{\Omega} (e^{4\pi u^2} - \beta u^2) dx$$

can be attained by some function $u \in W^{1,2}_0(\Omega)$ with $\int_{\Omega} |\nabla u|^2 dx - \alpha \int_{\Omega} u^2 dx = 1$. In the case $\beta = 0$, this is reduced to a result of Yang [24].


Keywords and phrases: Trudinger-Moser inequality, extremal function, blow-up analysis.

REFERENCES