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ON A GENERAL INEQUALITY RELATED TO THE GENERALIZED–EULER–CONSTANT FUNCTION

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Abstract. Let $\gamma(z)$ be the generalized-Euler-constant function. In this paper, we establish a general inequality related to $\gamma(z)$, which contains a result due to Chen and Han as a special case. We also obtain an inequality for the generalized Somos recurrence constant, using its relation with the generalized-Euler-constant function.

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