EXPONENTIAL TYPE LOCALLY GENERALIZED STRICTLY DOUBLE DIAGONALLY TENSORS AND EIGENVALUE LOCALIZATION

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Abstract. In this paper, we introduce exponential type locally generalized strictly double diagonally dominant tensors. This concept extends the concept of strictly diagonally dominant tensors. It is shown that exponential type locally generalized strictly double diagonally dominant tensors must be H-tensors. Furthermore, as applications of exponential type locally generalized strictly double diagonally dominant tensors, we present some new eigenvalue localization sets and checkable sufficient condition for the positive definiteness of even-order real symmetric tensors. Appropriate numerical examples are proposed to illustrate that our new tensors eigenvalue localization sets are more precise than some existing sets in some cases.

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