SHARP WILKER AND HUYGENS TYPE INEQUALITIES FOR TRIGONOMETRIC AND INVERSE TRIGONOMETRIC FUNCTIONS

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Abstract. In this paper, we prove Wilker and Huygens type inequalities for inverse trigonometric functions. This solves two conjectures proposed by Chao-Ping Chen. Also, we present new sharp Wilker and Huygens type inequalities for trigonometric functions.

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REFERENCES

[1] G. D. ANDERSON, S.-L. QIU, M. K. VAMANAMURTHY AND M. VUORINEN, *Generalized elliptic integral and modular equations*, Pacific J. Math. **192** (2000), 1–37.

[2] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Conformal Invariants, Inequalities, and Quasiconformal Maps*, New York, 1997.

[3] G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN, *Monotonicity of Some Functions in Calculus*, https://www.math.auckland.ac.nz/Research/Reports/Series/538.pdf.

[4] A. BARICZ AND J. SÁNDOR, *Extensions of generalized Wilker inequality to Bessel functions*, J. Math. Inequal. **2** (2008), 397–406.

[5] C.-P. CHEN, *Sharp Wilker- and Huygens-type inequalities for inverse trigonometric and inverse hyperbolic functions*, Integral Transforms Spec. Funct. **23**, 12 (2012), 865–873.

[6] C. P. CHEN AND W.S. CHEUNG, *Sharpness of Wilker and Huygens type inequalities*, J. Inequal. Appl. **2012**, 72,
http://www.journalofinequalitiesandapplications.com/content/2012/1/72.

[7] C. P. CHEN AND W. S. CHEUNG, *Inequalities and solution to Oppenheim's problem*, Integral Transforms Spec. Funct. **23**, 5 (2012), 325–336.

[8] C. P. CHEN AND J. SÁNDOR, *Inequality chains for Wilker, Huygens and Lazarević type inequalities*, J. Math. Inequal. **8**, 1 (2014), 55–67.

[9] B. N. GUO, B. M. QIAO, F. QI AND W. LI, *On new proofs of Wilker inequalities involving trigonometric functions*, Math. Inequal. Appl. **6** (2003), 19–22.

[10] C. HUYGENS, *Oeuvres Completes 1888-1940*, Société Hollondaise des Science, Haga.

[11] B. J. MALEŠEVIĆ, B. BANJAC AND I. JOVOVIĆ, *A proof of two conjectures of Chao-Ping Chen for inverse trigonometric functions*, J. Math. Inequal. **11**, 1 (2017), 151–162.

[12] D. S. MITRINOVIĆ, *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.

[13] C. MORTICI, *The natural approach of Wilker-Cusa-Huygens inequalities*, Math. Inequal. Appl. **14** (2011), 535–541.

[14] C. MORTICI, *A subtly analysis of Wilker inequality*, Appl. Math. Comput. **231** (2014), 516–520.

[15] E. NEUMAN, *One- and two-sided inequalities for Jacobian elliptic functions and related results*, Integral Transforms Spec. Funct. **21** (2010), 399–407.

[16] E. NEUMAN, *On Wilker and Huygens type inequalities*, Math. Inequal. Appl. **15** (2012), 271–279.

[17] E. NEUMAN, *Wilker and Huygens-type inequalities for the generalized trigonometric and for the generalized hyperbolic functions*, Appl. Math. Comput. **230** (2014), 211–217.

[18] E. NEUMAN AND J. SÁNDOR, *On some inequalities involving trigonometric and hyperbolic functions with emphasis on the Cusa-Huygens, Wilker, and Huygens inequalities*, Math. Inequal. Appl. **13** (2010), 715–723.

[19] I. PINELIS, *L'Hospital rules of monotonicity and Wilker-Anglesio inequality*, Amer. Math. Monthly, **111** (2004), 905–909.

[20] J. SÁNDOR AND M. BENCZE, *On Huygens' trigonometric inequality*, RGMIA Res. Rep. Collection **8**, 3 (2005), Article 14.

[21] R. SRIVASTAVA, *Some families of integral, trigonometric and other related inequalities*, Appl. Math. Inform. Sci. **5** (2011), 342–360.

[22] J. S. SUMNER, A. A. JAGERS, M. VOWE AND J. ANGLESIO, *Inequalities involving trigonometric functions*, Amer. Math. Monthly **98** (1991), 264–267.

[23] J. B. WILKER, *Problem E 3306*, Amer. Math. Monthly **96** (1989), 55.

[24] S. H. WU, *On extension and refinement of Wilker's inequality*, Rocky Mountain J. Math. **39** (2009), 683–687.

[25] S. H. WU AND A. BARICZ, *Generalizations of Mitrinović, Adamović and Lazarevic's inequalities and their applications*, Publ. Math. Debrecen **75** (2009), 447–458.

[26] S. H. WU AND H. M. SRIVASTAVA, *A weighted and exponential generalization of Wilker's inequality and its applications*, Integral Transforms and Spec. Funct. **18** (2007), 529–535.

[27] S. H. WU AND H. M. SRIVASTAVA, *A further refinement of Wilker's inequality*, Integral Transforms and Spec. Funct. **19** (2008), 757–765.

[28] L. ZHANG AND L. ZHU, *A new elementary proof of Wilker's inequalities*, Math. Inequal. Appl. **11** (2008), 149–151.

[29] L. ZHU, *A new simple proof of Wilker's inequality*, Math. Inequal. Appl. **8** (2005), 749–750.

[30] L. ZHU, *On Wilker-type inequalities*, Math. Inequal. Appl. **10** (2007), 727–731.

[31] L. ZHU, *Some new Wilker-type inequalities for circular and hyperbolic functions*, Abstr. Appl. Anal. **2009**, Article ID 485842.

[32] L. ZHU, *A source of inequalities for circular functions*, Comput. Math. Appl. **58** (2009), 1998–2004.

[33] L. ZHU, *Inequalities for Hyperbolic functions and their Applications*, J. Inequal. Appl. **2010**, Article ID 130821.